DO NOW

Turn in Quiz 3.4 Take Home Portion

3.5 Implicit Differentiation

Explicit Form - when function is expressed with "y" solved for in terms of "x"

Ex: $y = 4x^3 - 5x$

Implicit Form - when "y" is contained anywhere in the equation - on both sides

Ex: $2y^2 + 3xy = 7x - 9y$

Our derivatives so far have been of functions in which form?

Explicit form

What if the function is in Implicit Form?

**To find $\frac{dy}{dx}$ implicitly, you must realize that differentiation is taking place: with respect to x.

-To differentiate terms involving only x:

differentiate as usual.

-To differentiate terms involving other variables, you must apply: the chain rule

Examples:

a. $\frac{d}{dx}[x^4] = 4x^3$ (match)

b. $\frac{d}{dx}[y^4] = 4y^3 \cdot y'$ (Don't match) $\quad 4y^3 \cdot \frac{dy}{dx}$

c. $\frac{d}{dx}[2x + 4y]$

$2 + 4\frac{dy}{dx}$

d. $\frac{d}{dx}[x^2y]$

$x^2 \cdot 1y' + y\,2x$

$x^2y' + 2xy$

$x^2\frac{dy}{dx} + 2xy$

Guidelines for Implicit Differentiation:

1. Differentiate both sides of the equation with respect to x.
   * use $\frac{dy}{dx}$ or y' when necessary
2. Collect all terms with $\frac{dy}{dx}$ (or y') on the left side of the equation and those without on the right.
3. Factor $\frac{dy}{dx}$ (or y') out of the left side.
4. Solve for $\frac{dy}{dx}$ (or y') using division.

1. Differentiate the function: $2x^3 - 4y + 3 = 0$

Explicitly: Solve for y first

$-4y = -2x^3 - 3$

$y = \frac{1}{2}x^3 + \frac{3}{4}$

$\boxed{y' = \frac{3}{2}x^2}$

Implicitly: $2x^3 - 4y + 3 = 0$

$6x^2 - 4\frac{dy}{dx} = 0$

$-4\frac{dy}{dx} = -6x^2$

$\frac{dy}{dx} = \frac{-6x^2}{-4}$

$\boxed{\frac{dy}{dx} = \frac{3}{2}x^2}$

2. Find y' given that $y^3 + y^2 - 5y - x^2 = -4$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(3y^2 + 2y - 5) = 2x$$

$$y' = \frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

* Notice the derivative now contains both x and y

$y^3 + y^2 - 5y - x^2 = -4$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

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Find the slope of the tangent line at the following points:

a. (2, 0)

$$m = \frac{2(2)}{3(0)^2 + 2(0) - 5}$$

$$m = -\frac{4}{5}$$

b. (1, 1)

$$m = \frac{2(1)}{3(1)^2 + 2(1) - 5}$$

$$m = \frac{2}{0}$$

undefined

c. (1, -3)

$$m = \frac{2(1)}{3(-3)^2 + 2(-3) - 5}$$

$$m = \frac{2}{27 - 6 - 5}$$

$$m = \frac{1}{8}$$

Find the derivative.

4. $x^4 - x^2y^2 = y^2$

$$4x^3 - x^2 \cdot 2y \cdot y' - y^2 \cdot 2x = 2y \cdot y'$$

$$-x^2 2y\, y' - 2y y' = y^2 \cdot 2x - 4x^3$$

$$y'(-x^2 2y - 2y) = 2xy^2 - 4x^3$$

$$y' = \frac{2xy^2 - 4x^3}{-x^2 2y - 2y}$$

$$y' = \frac{-2(2x^3 - xy^2)}{-2(x^2 y + y)}$$

$$y' = \frac{2x^3 - xy^2}{x^2 y + y}$$

5. $x = \sin y^3$

$$1 = \cos y^3 \cdot 3y^2 \cdot \frac{dy}{dx}$$

$$\frac{1}{3y^2 \cos y^3} = \frac{dy}{dx}$$

$z = \sin u$

$u = y^3$

6. $\ln(xy) + 5x = 30$

$$\ln x + \ln y + 5x = 30$$

$$\frac{1}{x} + \frac{1}{y} \cdot y' + 5 = 0$$

$$\frac{y'}{y} = -\frac{1}{x} - 5$$

$$y' = y\left(-\frac{1}{x} - 5\right)$$

$$y' = -\left(\frac{y}{x} + 5y\right)$$

$$y' = -\left(\frac{y + 5xy}{x}\right)$$

HOMEWORK

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